5. (a) Prove that : **3**

$$H_n(x) = 2^n \left[\exp\left(\frac{-1}{4} \frac{d^2}{dx^2} \right) \right] x^n$$

(b) Show that : **2**

$$H_6(x) = 64x^6 + 480x^4 + 720x^2 - 120$$

Section III

6. (a) Find Lapalce transform of $\sin h3t\cos^2 t$.

(b) Find Laplace transform of

$$f(x) = \begin{cases} 0 & 0 < t < 1 \\ t & 1 < t < 2 \\ 0 & t > 2 \end{cases}.$$ **2**

7. (a) Find Inverse Laplace transform of $\tan^{-1}\left(\frac{2}{5^2}\right)$. **3**

(b) Solve : **2**

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{-x}, \ y(0) = 1, \ y'(0) = 1$$

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B. A. & Hons. (Subsidiary) EXAMINATION, 2025

(Fourth Semester)

(Re-appear Only)

MATHEMATICS

Paper BM-242

Special Functions & Integral Transforms

Time : 3 Hours] [*Maximum Marks :* 26

Before answering the question-paper, candidates must ensure that they have been supplied with correct and complete question-paper. No complaint, in this regard will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

1. (a) Write solution of : **1**

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}+xy=0$$

(b) Show that $x = 0$ is ordinary point of differential equation : **1**

$$\left(x^2+1\right)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-xy=0$$

(c) Find Laplace transform of $\sinh\left(\frac{t}{2}\right).\sin\frac{\sqrt{3}t}{2}$. **2**

(d) Prove :

$$\int_{-1}^{1}\left(1-x^2\right)P_m'(x)P_n'(x)\,dx=0$$

when $m \neq n$. **2**

Section I

2. (a) Find the series solution of following equation about '0' : **3**

$$x\frac{d^2y}{dx^2}-3\frac{dy}{dx}+xy=0$$

(b) Find power series solution of : **2**

$$\left(1-x^2\right)\frac{d^2y}{dx^2}+2y=0,\quad y(0)=4,\ y'(0)=5$$

3. (a) Show that $x^n J_n(x)$ is solution of : 3

$$(x)\frac{d^2y}{dx^2}+(1-2n)\frac{dy}{dx}+xy=0$$

(b) Solve :

$$9x^2\frac{d^2y}{dx^2}+9x\frac{dy}{dx}+\left(36x^4-16\right)y=0$$

by using substitution $z=x^2$. **2**

Section II

4. (a) Show that : **3**

$$\left(1-2xt+t^2\right)^{-1/2}=\sum_{n=0}^{\infty}t_nP_n(x)\quad |x|\le 1,\ |t|\le 1$$

(b) Solve by Legendre's equation (when $n = 0$) : **2**

$$\left(1-x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}=0$$

Section IV

8. (a) Find the fourier sine transform of $\frac{e^{-ax}}{x}$. **2**

(b) Using Parseval's Identity show that : **3**

$$\int_0^\infty \frac{dx}{(x^2+25)(x^2+81)} = \frac{\pi}{1260}$$

9. (a) Verify Convolution theorem for : **2**

$$f(x) = g(x) = e^{-x^2}$$

(b) Find the finite cosine transform of : **3**

$$f(x) = \sin ax$$

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